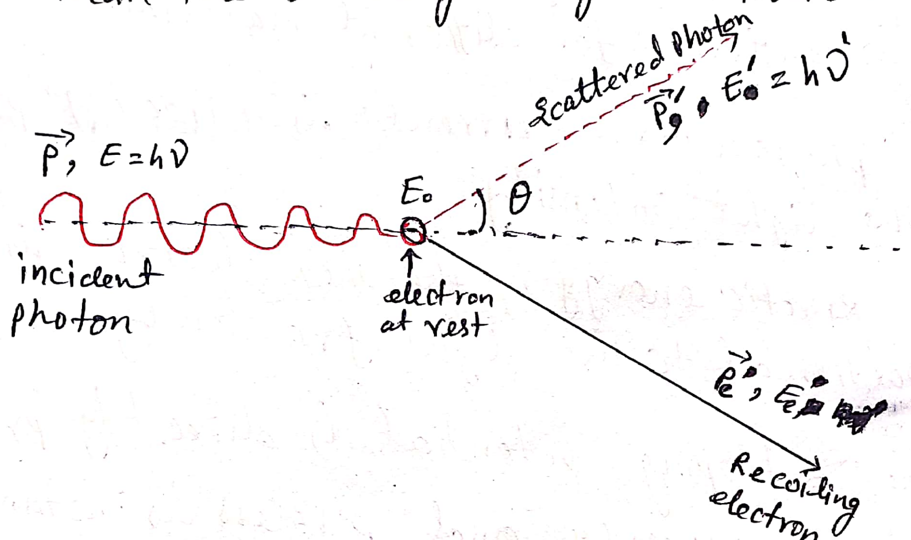
Compton Effect :-> Compton provided the information of the particle aspect of radiation. He found that the wavelength of the scattered radiation is larger than the wavelength of the incident radiation.



Consider that the incident photon of energy $E = h\nu$ and momentum $p = \frac{hc}{\nu}$ collides with an electron that is initially at rest. If the photon scattered with momentum $\vec{p}'$ at angle 'θ' while the electron recoils with momentum $\vec{P_e}$.

Now by the conservation of linear momentum

$$\vec{P} = \vec{P_e} + \vec{P}'$$

$$\vec{P_e} = \vec{P} - \vec{P}'$$

taking square in both side,

$$P_e^2 = (\vec{P} - \vec{P}')^2$$

$$P_e^2 = P^2 + P'^2 - 2|\vec{P}||\vec{P}'|\cos\theta$$

$$P_e^2 = p^2 + p'^2 - 2pp' \cos\theta.$$

$$p = \frac{h\nu}{c} \quad , \quad p' = \frac{h\nu'}{c}$$

$$P_e^2 = \frac{h^2}{c^2}\left(\nu^2 + \nu'^2 - 2\nu\nu' \cos\theta\right) \quad \text{——} (1)$$

Now by the law of conservation of energy under elastic collision.

$$E + E_0 = E_e + E'$$

$$E_0 = m_e c^2$$

$$E_e = \sqrt{P_e^2 c^2 + m_e^2 c^4}$$

$$E = h\nu$$

$$E' = h\nu'$$

Now

$$h\nu + m_e c^2 = \sqrt{P_e^2 c^2 + m_e^2 c^4} + h\nu'$$

Put the value of P_e^2 from equation (1)

$$h\nu + m_e c^2 = \sqrt{\frac{h^2}{\cancel{c^2}}\left(\nu^2 + \nu'^2 - 2\nu\nu' \cos\theta\right)\cancel{c^2} + m_e^2 c^4} + h\nu'$$

$$h\nu + m_e c^2 = h\sqrt{\left(\nu^2 + \nu'^2 - 2\nu\nu' \cos\theta\right) + \frac{m_e^2 c^4}{h^2}} + h\nu'$$

$$h\nu - h\nu' + m_e c^2 = h\sqrt{\left(\nu^2 + \nu'^2 - 2\nu\nu' \cos\theta\right) + \frac{m_e^2 c^4}{h^2}}$$

$$\nu - \nu' + \frac{m_e c^2}{h} = \sqrt{\left(\nu^2 + \nu'^2\right) - 2\nu\nu' \cos\theta) + \frac{m_e^2 c^4}{h^2}}$$

$$(\nu-\nu') + \frac{m_e c^2}{h} = \sqrt{\nu^2+\nu'^2 - 2\nu\nu'\cos\theta + \frac{m_e^2 c^4}{h^2}}$$

Squaring both the side.

$$(\nu-\nu')^2 + \left(\frac{m_e c^2}{h}\right)^2 + 2(\nu-\nu')\frac{m_e c^2}{h} = (\nu^2+\nu'^2 - 2\nu\nu'\cos\theta) + \frac{m_e^2 c^4}{h^2}$$

$$\cancel{\nu^2} + \cancel{\nu'^2} - 2\nu\nu' + \cancel{\frac{m_e^2 c^4}{h^2}} + 2(\nu-\nu')\frac{m_e c^2}{h} = \cancel{\nu^2} + \cancel{\nu'^2} - 2\nu\nu'\cos\theta + \cancel{\frac{m_e^2 c^4}{h^2}}$$

$$2(\nu-\nu')\frac{m_e c^2}{h} = 2\nu\nu' - 2\nu\nu'\cos\theta.$$

$$\cancel{2}(\nu-\nu')\frac{m_e c^2}{h} = \cancel{2}\nu\nu'(1-\cos\theta)$$

$$\left(\frac{\cancel{\nu}}{\cancel{\nu}\nu'} - \frac{\cancel{\nu'}}{\nu\cancel{\nu'}}\right)\frac{m_e c^2}{h} = (1-\cos\theta)$$

$$\left(\frac{1}{\nu'} - \frac{1}{\nu}\right) = \frac{h}{m_e c^2}(1-\cos\theta)$$

$$\nu = \frac{c}{\lambda}, \quad \nu' = \frac{c}{\lambda'}$$

$$\lambda = \frac{c}{\nu} \Rightarrow \frac{\lambda}{c} = \frac{1}{\nu}, \quad \frac{\lambda'}{c} = \frac{1}{\nu'}$$

$$\left(\frac{\lambda'}{c} - \frac{\lambda}{c}\right) = \frac{h}{m_e c^2}(1-\cos\theta)$$

$$\frac{1}{\cancel{c}}(\lambda'-\lambda) = \frac{h}{m_e c \times \cancel{c}}(1-\cos\theta)$$

$$\boxed{(\lambda'-\lambda) = \frac{h}{m_e c}(1-\cos\theta)}$$

The wavelength shift is

$$\boxed{\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)}$$

The compton effect confirms that photons behaves like a particle.

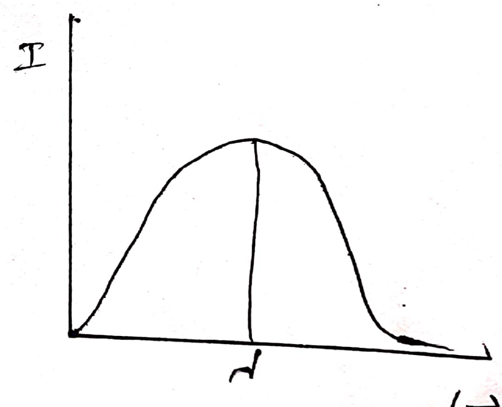
Case.

$$\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$$

if $\theta = 0^\circ$, $\cos 0^\circ = 1$.

$$\lambda' - \lambda = 0$$

$$\boxed{\lambda' = \lambda}$$



if $\theta = 90^\circ$, $\cos 90^\circ = 0$

$$\lambda' - \lambda = \frac{h}{m_e c}$$

$$\boxed{\lambda' = \lambda + \frac{h}{m_e c}}$$